Cluster structure of a low-energy resonance in tetraneutron

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We theoretically investigate the possibility for a tetraneutron to exist as a low-energy resonance state. We explore a microscopic model based on the assumption that the tetraneutron can be treated as a compound system where ^{3}n+n and ^{2}n+^{2}n coupled cluster configurations coexist. The influence of the Pauli principle on the kinetic energy of the relative motion of the neutron clusters is shown to result in their attraction. The strength of such attraction is high enough to ensure the existence of a low-energy resonance in the tetraneutron, provided that the oscillator length is large enough.

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The first claim of the experimental observation of the nuclear stable tetraneutron has been made in [1]. Since then all other experimental attempts to find either a bound or a resonance state in the system of four neutrons have not met with success. However, in a recently reported experiment with the breakup of ^{14}Be [2] six events consistent with the formation of a bound tetraneutron were revealed. Unfortunately, several other experiments [3, 4, 5] undertaken to verify these results failed to prove or refute completely the existence of the tetraneutron due to a poor statistics. An overall conclusion of a number of theoretical papers on this subject [6, 7, 8, 9] is that it does not seem possible to change modern nuclear Hamiltonians to bind a tetraneutron without destroying many other successful predictions of those Hamiltonians. For instance, calculations within the hyperspherical functions method (HSFM) [7] suggest that a very strong phenomenological four-nucleon force is needed in order to bind the tetraneutron. And yet neither theoretical nor experimental results exclude the possible existence of tetraneutron as a low-energy resonance (see [6, 9, 10]).

There are only few cases of theoretical treatment of the resonant tetraneutron. In Ref. [9] the continuum states of the ^{4}n system were studied in the framework of the approach which combines concepts of the HSFM and the resonating-group method (RGM). Along with the lowest order hyperharmonic the authors of Ref. [9] invoked the hyperharmonics with hypermomenta $K = K_{\min} + 2$ and $K = K_{\min} + 4$, which reproduce ^{2}n+^{2}n clustering of the tetraneutron. The analysis of the energy behaviour of the eigenphases led authors to the conclusion that ^{4}n has a resonance state at an energy of about 1-3 MeV. But clear indication of such a resonance was obtained only for the Volkov effective NN potential, which is known to be inappropriate for studying multineutron systems as it binds dineutron.

The most systematic study of four-neutron resonances was performed in [8], where configuration space Faddeev-Yakubovsky equations have been solved using realistic NN interaction to follow the resonance pole trajectories in the complex energy plane. It was concluded in [8] that tetraneutron – bound or resonant – can be created only in strong external fields and would disintegrate right after such a field is removed. The same authors remarked, however, that an accurate determination of the physical resonance position was not possible with the methods used in [8].

Hence, the existence of resonant tetraneutrons is still an open question. In the present paper, we study the possibility for the tetraneutron to be described as a compound system where ^{3}n+n and ^{2}n+^{2}n coupled cluster configurations coexist.

The Pauli exclusion principle is known to significantly influence the interaction of composite particles. An exact treatment of the antisymmetrization effects related to the kinetic energy exclusively was shown to result in either an effective repulsion or attraction of the clusters [11]. Such an effective interaction substantially affects the dynamics of the cluster-cluster interaction and can, on occasion, produce resonance behaviour of the scattering phase or even a bound state in compound nuclear system. The role of the Pauli principle in the formation of both the discrete spectrum and multi-channel states of the two-cluster nuclear systems can be studied in the Algebraic Version of the resonating-group method (AVRGM) [12]. An RGM wave function is built in the form of an antisymmetrized product of cluster internal wave functions [17] and a wave function of their relative motion. The latter depends only on the Jacobi vector $\mathbf{R}$ of the considered binary system and, according to the AVRGM, is expanded in a complete discrete basis of harmonic-oscillator (HO) states allowed by the Pauli principle

$$\Psi_{\kappa\,(E)}(\mathbf{R}) = \sum_n \sqrt{\Lambda_n} C_n^{\kappa\,(E)} \psi_n(\mathbf{R}).$$

The Pauli-allowed basis functions ψ_n are defined in the Fock-Bargmann representation [13], where they take exceptionally simple form. Expansion coefficients both of the discrete eigenstates with energy $E_\kappa = -\kappa^2/2 < 0$

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and of the continuum eigenstates $\{C_n(E)\}$ with energy $E > 0$ are found by solving a set of linear equations

$$\sum_{n'} \langle n|\hat{H}|n'\rangle C_{n'} - E\Lambda_n C_n = 0. \quad (1)$$

The asymptotic behavior of the continuum eigenstates is expressed in terms of Hankel functions of the first and second kind, and the scattering S-matrix elements.

The set of quantum numbers n includes the number of oscillator quanta ν, the indices (λ, μ) of their SU(3) symmetry, the additional quantum number $\alpha_{(\lambda,\mu)}$ if there are several differing (λ, μ) multiplets, the orbital momentum L and its projection M, and one more additional quantum number α_L if the multiplet (λ, μ) has several states with the same values of L. In this paper we restrict the discussion to the case of $L = M = 0$ and, hence, we will use only first three quantum numbers.

The influence of the Pauli exclusion principle on the collision of clusters through the kinetic energy is determined by the eigenvalues Λ_ν of the antisymmetrizer:

$$\hat{A}\psi_\nu = \Lambda_\nu \psi_\nu, \quad \Lambda_\nu \geq 0, \quad \Lambda_\nu \to 1, \nu \to \infty.$$

The eigenvalues are proportional to the probability of the system being in the state determined by the eigenfunction ψ_ν. $\Lambda_\nu > 0$ correspond to the Pauli-allowed states, while zero eigenvalues belong to the Pauli-forbidden states. $\Lambda_\nu < 1$ generate repulsion of clusters, whereas an attraction appears in the states with the eigenvalues exceeding unity.

Due to the exchange of nucleons belonging to different clusters, Eq. (1) resembles the Schrödinger equation for relative motion of two particles with highly nonlocal interaction between them. This complicates an analysis of the cluster-cluster interaction generated by the nucleon-nucleon forces. However, if the oscillator length r_0 is much less than the range b_0 of the NN potential, then the potential energy matrix in the HO representation is equivalent to the diagonal matrix which is a discrete analog of cluster-cluster potential in the coordinate space

$$\sum_{\nu'_{\min}}^{\infty} \langle \nu, l|U|\nu', l\rangle C_{\nu',l} = U_{\text{eff}}(r_\nu) C_{\nu,l}. \quad (2)$$

For a central nucleon-nucleon potential having a Gaussian form we will have

$$U_{\text{eff}}(r_\nu) \sim U_0 \exp\left\{-\frac{r_\nu^2}{b_0^2}\right\}, r_\nu = \sqrt{\frac{A_1 + A_2}{A_1 A_2}} r_0 \sqrt{2\nu + 2l + 3}, \quad (3)$$

where U_0 is the strength of the Gaussian potential, r_ν defines the distance between clusters (composed of A_1 and A_2 nucleons, respectively) and l is the angular momentum of the cluster relative motion.

The validity of relation (2) is demonstrated in Fig. 1 by the example of effective ^2n-^2n potential generated by the Serber NN interaction [14] ($r_0 = 0.15$ fm, b_0=1.48 fm, $\nu = 2k$). So, we deduce from (3) that the effective

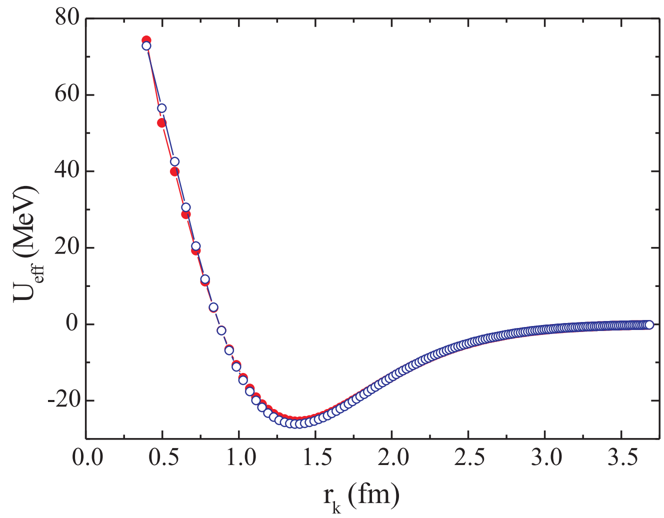

FIG. 1: (Color online) Effective ^2n-^2n potential generated by the Serber interaction. Solid circles: $\sum_{k'=1}^{\infty}\langle k|U|k'\rangle$. Empty circles: equivalent local potential $U_{\text{eff}}(r_k)$ (see text for details).

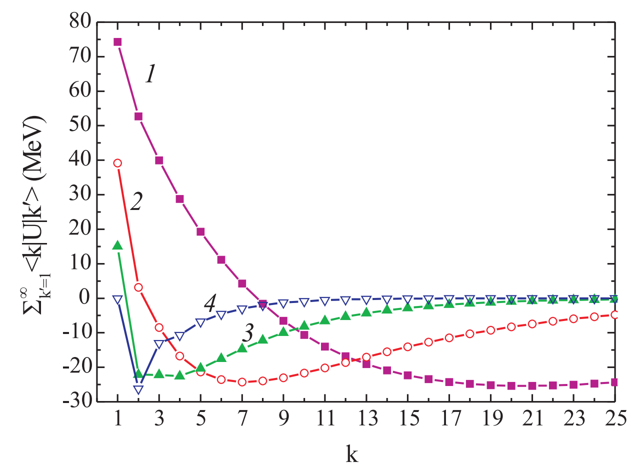

FIG. 2: (Color online) Effective ^2n-^2n potential generated by the Serber interaction versus the number of quanta k at different values of oscillator length r_0. Curves: *1* – $r_0 = 0.15$ fm; *2* – $r_0 = 0.25$ fm; *3* – $r_0 = 0.35$ fm; *4* – $r_0 = 0.5$ fm.

cluster-cluster potential decreases exponentially as oscillator length r_0 increases (see Fig. 2). It would appear natural that r_0 should be large enough to give a reasonable fit to the neutron density distribution inside dineutron clusters, i.e. $r_0 > b_0$. But then we immediately arrive at the conclusion that *effective intercluster interaction derived from the NN potential is of minor importance and can not produce a resonance in ^4n system.* We have used the Gaussian-type potential for simplicity, but all the above is certain to remain valid for any short-range potential.

As regards the contribution from the kinetic energy

operator (with its exchange part), in this case it is reduced to the elimination of a single Pauli forbidden state by virtue of the fact that for the $^2n+^2n$ configuration the eigenvalues of all the allowed states equal to unity. Thus, the eigenfunctions of the kinetic energy operator in the region of relatively small energies are similar to the eigenfunctions of a particle in the field of a hard core, and, in the region of large energies, they are identical to the wave functions of free motion of a particle. Hence, there is no grounds to believe that the tetraneutron can exist as a dineutron-dineutron cluster system. Behaviour of the phase shift of the $^2n+^2n$ scattering, shown in Fig. 3, supports this inference. Figure 3 also corroborates our

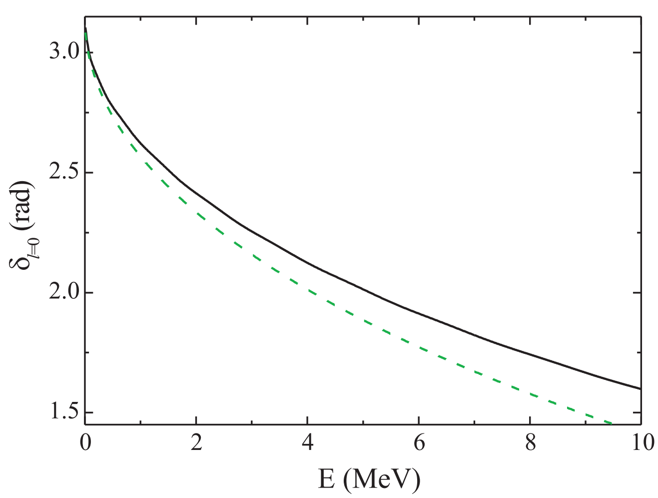

FIG. 3: (Color online) Phase shift of the $^2n+^2n$ scattering, $r_0 = b_0 = 1.48$ fm. Dashed curve: the phase obtained by incorporating the kinetic energy exchange potential exclusively. Solid curve: phase shift calculated with the inclusion of the cluster-cluster potential generated by Ripka forces [15].

conclusion that the ^{2}n-^{2}n interaction generated by the NN forces has a minor effect on physical observables [18].

For each compound nuclear system, one can identify several cluster configurations corresponding to a certain set of clusters released as a result of a nuclear reaction. The probability for a cluster configuration to be realized in the Pauli-allowed basis function of a binary cluster system with the minimum number of quanta was shown to be proportional to the eigenvalue of isolated configuration [16]. Maximum eigenvalues belong to the ^{3}n+n configuration of the ^{4}n system. They exceed unity and have a maximum at the minimum number of quanta that can be interpreted as an attraction of neutron clusters at small distance between them. Taking into account various cluster configurations of the compound nucleus causes the maximum eigenvalues to increase. Moreover, new branches of excitation appear with particularly large (larger than unity) eigenvalues of the allowed states that is an evidence of essential attraction of the clusters due to exchange effects. Therefore, we have good reason to believe that consideration of $^2n+^2n$ and ^3n+n coupled cluster configurations of the ^4n system will allow us to solve the problem of existence of a bound state or a resonance in the tetraneutron.

The $^2n+^2n$ and the ^3n+n configuration coupled provide two branches of the two-fold degenerate SU(3) representation $(2k,0)$. The behaviour of the eigenvalues belonging to these branches is shown in Fig. 4. An effective attraction reveals itself in $(2k,0)_+$-branch while in $(2k,0)_-$-branch an effective repulsion takes place. The

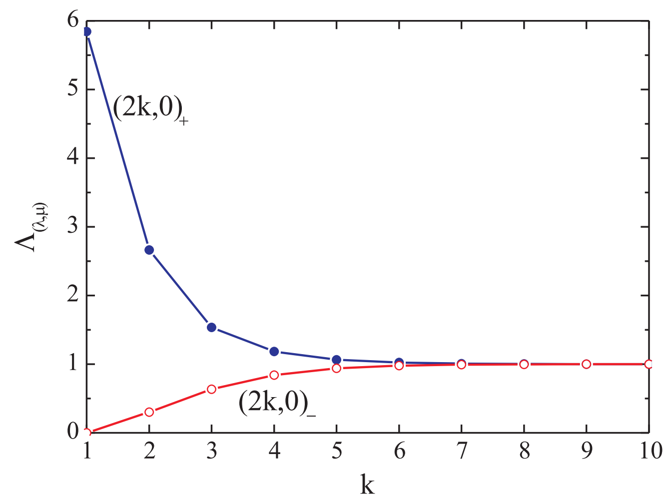

FIG. 4: (Color online) Eigenvalues $\Lambda_{(\lambda,\mu)}$ of the allowed states for the ^4n system in the coupled-channel approach.

remarkable feature of these branches is that at large distance between clusters ($k \to \infty$) they contain the wave function of both cluster configurations on equal terms:

$$\Psi_{(2k,0)_\pm} \to \frac{1}{\sqrt{2}}\left\{\psi_{(2k,0)}(^3n+n) \pm \bar{\psi}_{(2k,0)}(^2n+^2n)\right\}.$$

Returning to the analysis of intercluster interaction generated by NN forces, we refer to Fig. 5. Figure 5 depicts the effective cluster-cluster potentials generated by the Ripka forces and calculated for isolated cluster configurations of the ^{4}n system. It is seen from Fig. 5 that the strength of ^{3}n-n interaction exceeds that of ^{2}n-^{2}n interaction by the difference Δ_u in intrinsic potential energy of the ^{3}n cluster and two dineutrons. For the Gaussian-type potential $\Delta_u = -0.5V_{13}(1+2r_0^2/b_0^2)^{-3/2}$, where V_{13} is the strength of interaction of a neutron pair in singlet spin state.

The Δ_u causes the SU(3)-branches $(2k,0)_+$ and $(2k,0)_-$ to remain coupled even at large k (see Fig. 6). The difference in intrinsic potential energies Δ_u of ^3n and two dineutrons, along with the difference in their kinetic energies Δ_t, gives rise to the threshold of the tetraneutron decay into ^3n and a neutron. Although the strength of such energy barrier $\Delta = \Delta_u + \Delta_t$ is mainly determined by the discrepancy between intrinsic kinetic energies of the clusters [19], the latter does not couple basis functions belonging to different SU(3) branches. At the same

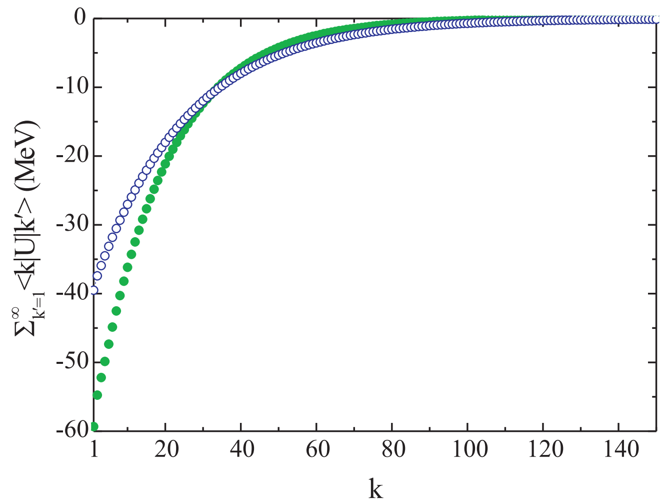

FIG. 5: (Color online) Effective intercluster potential induced by the Ripka interaction. Solid and empty circles: ^{3}n+n and ^{2}n+^{2}n cluster configurations, correspondingly.

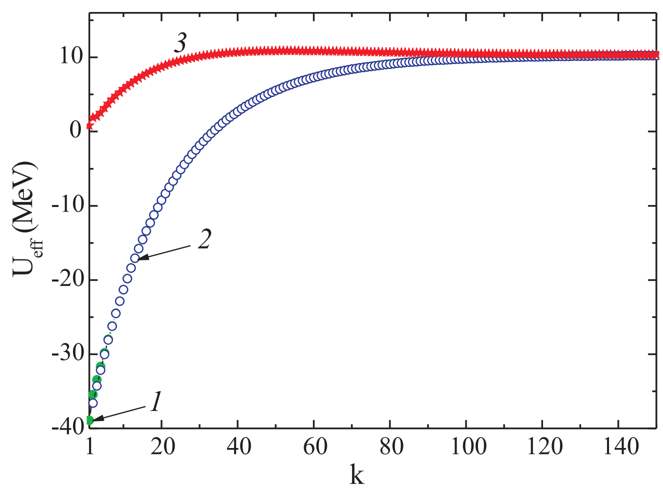

FIG. 6: (Color online) Effective cluster-cluster potential $U_k^{\rm eff}$ generated by Ripka interaction for the ^4n system in the coupled-channel approach. Curves: *1* – $U_k^{++} = \sum_{k'=1}^{\infty}\langle(2k,0)_+|U|(2k',0)_+\rangle$; *2* – $U_k^{--} = \sum_{k'=1}^{\infty}\langle(2k,0)_-|U|(2k',0)_-\rangle$; *3* – $U_k^{+-} = \sum_{k'=1}^{\infty}\langle(2k,0)_+|U|(2k',0)_-\rangle$.

time, nonzero Δ_u implies additional repulsion, which prevents the formation of resonance in the tetraneutron. Δ_u tends to diminish with increasing the oscillator length r_0. Hence, *effective attraction generated by the kinetic exchange terms is able to create a low-energy resonance in* ^4n *only at large values of* r_0. Figure 7, which represents the phase shift of the elastic $^2n +^2 n$ scattering versus dimensionless energy, sustains this assumption. A resonance behaviour of the phase shift is observed only at the values of r_0 of the order of 10 fm. This resonance is very close to zero energy above the ^{4}n$\rightarrow^2$n+^2n threshold, and its width is quite large in comparison with its energy.

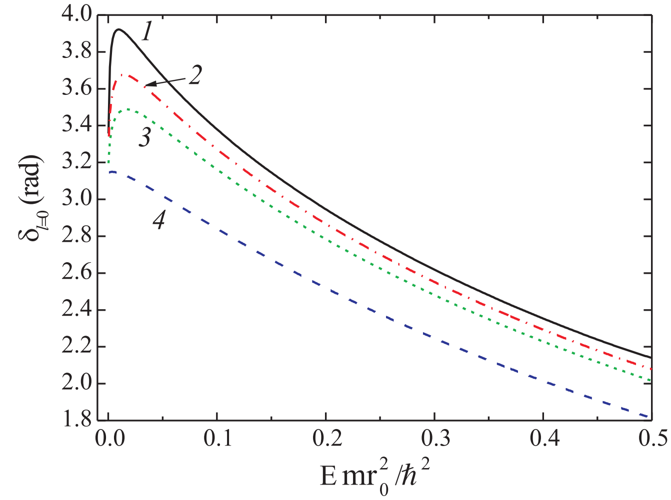

FIG. 7: (Color online) Phase shift of the elastic $^2n +^2 n$ scattering in the coupled-channel approach, provided that $\Delta_u = 0$ (*1*) and $\Delta_u \neq 0$ (produced by Ripka nucleon-nucleon forces of the range $b = 1.48$ fm) at $r_0 = 10b$ (*2*); $r_0 = 5b$ (*3*) and $r_0 = b$ (*4*).

Hence, our results are consistent with the experimental data [2]. Indeed, recently the authors of the latter paper discussed the description of these data by means of an unbound-tetraneutron resonance in [10]. They came to recognize that a broad low-energy resonance in the tetraneutron can be compatible with the events observed in [2]. Moreover, theoretical *ab initio* calculations of Pieper [6] also leaves room for the occurrence of a broad ^{4}n resonance with an energy of around 2 MeV or less.

In conclusion, we revealed that the tetraneutron has a good chance to exist as a compound system where ^{3}n+n and ^{2}n+^{2}n coupled cluster configurations coexist. The influence of the Pauli principle on the kinetic energy of the relative motion of the neutron clusters was shown to result in their attraction. The strength of such attraction is high enough to ensure the existence of a low-energy resonance in the tetraneutron, provided that the oscillator length is large enough. It was also demonstrated that increasing the oscillator length results in a depression of the cluster-cluster potential. For this reason the results are not sensitive to the choice of the phenomenological two-body nucleon-nucleon potential. No three- and four-nucleon forces are employed in the calculations.

We believe that our theoretical predictions can provide fresh insight into the problem of existence of resonance states in pure neutron systems and may help to illuminate the nature of such states and mechanism of their formation.

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[17] They are assumed to be fixed and described by the translation-invariant shell model wave functions with the same oscillator length r_0 for both clusters.
[18] We used the Ripka NN force to accentuate that even purely attractive NN potential is not able to contribute significantly to the scattering phase.
[19] $\Delta_t = 0.5\hbar^2/mr_0^2$, m denotes nucleon mass.